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Hidden scales in statistics of citation indicators

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ABSTRACT

Scholarly citations – widely seen as tangible measures of the impact and significance of academic papers – guide critical decisions by research administrators and policy makers. The citation distributions form characteristic patterns that can be revealed by big-data analysis. However, the citation dynamics varies significantly among subject areas, countries *etc*. The problem is how to quantify those differences, separate global and local citation characteristics. Here, we carry out an extensive analysis of the power-law relationship between the total citation count and the *h*-index to detect a functional dependence among its parameters for different science domains. The results demonstrate that the statistical structure of the citation indicators admits representation by a global scale and a set of local exponents. The scale parameters are evaluated for different research actors – individual researchers and entire countries – employing subject- and affiliation-based divisions of science into domains. The results can inform research assessment and classification into subject areas; the proposed divide-and-conquer approach can be applied to hidden scales in other power-law systems.

1. Introduction

The science of science strives for universal laws to understand and predict research processes and their outcomes (Zeng et al., 2017; Fortunato et al., 2018). In this endeavor, citation of an academic paper serves as an elementary unit of research recognition. Although reasons for citing are numerous, citation counts govern research policies, allocation of research resources, and hiring decisions. In this regard, the knowledge of regular patterns in the citation process is particularly helpful. To give a few examples, citation analysis provides insights into long-term scientific impact (Wang et al., 2013), evolution of research fields (Battiston et al., 2019) and individual career paths (Petersen et al., 2012; Sinatra et al., 2016). It is remarkable that the patterns extend beyond scientific careers, to other creative domains (Liu et al., 2021). One of the main features of science is that it is highly inhomogeneous, exhibiting significant differences among subject areas, countries *etc.* Accounting for these differences is important, for example, to make a fair comparison of scholars from different backgrounds. Extensive citation data are becoming increasingly available providing thus an opportunity to perform empirical citation analysis on different scales.

The hierarchical structure of science suggests potential variations in citation statistics of two principal types. The first type is size-related – citation metrics can be applied not only to individual researchers but also to larger, composite research actors such as institutions (Abramo et al., 2013) and countries (Csajbók et al., 2007). The other type corresponds to variations in publication and citation patterns across scientific disciplines; it causes a bias in application of citation-based metrics. The problem is tackled by normalization of impact metrics to mitigate the discipline bias (Kaur et al., 2013; Radicchi et al., 2008). In particular, it has been suggested that scaling citation distributions by the average number of citations per article results in a universal curve (Radicchi et al., 2008); however, significant exceptions have been found (Waltman et al., 2011). The general problem can be formulated as separation of global and local variables in the citation dynamics. Our empirical solution is to take some citation law (a relationship between

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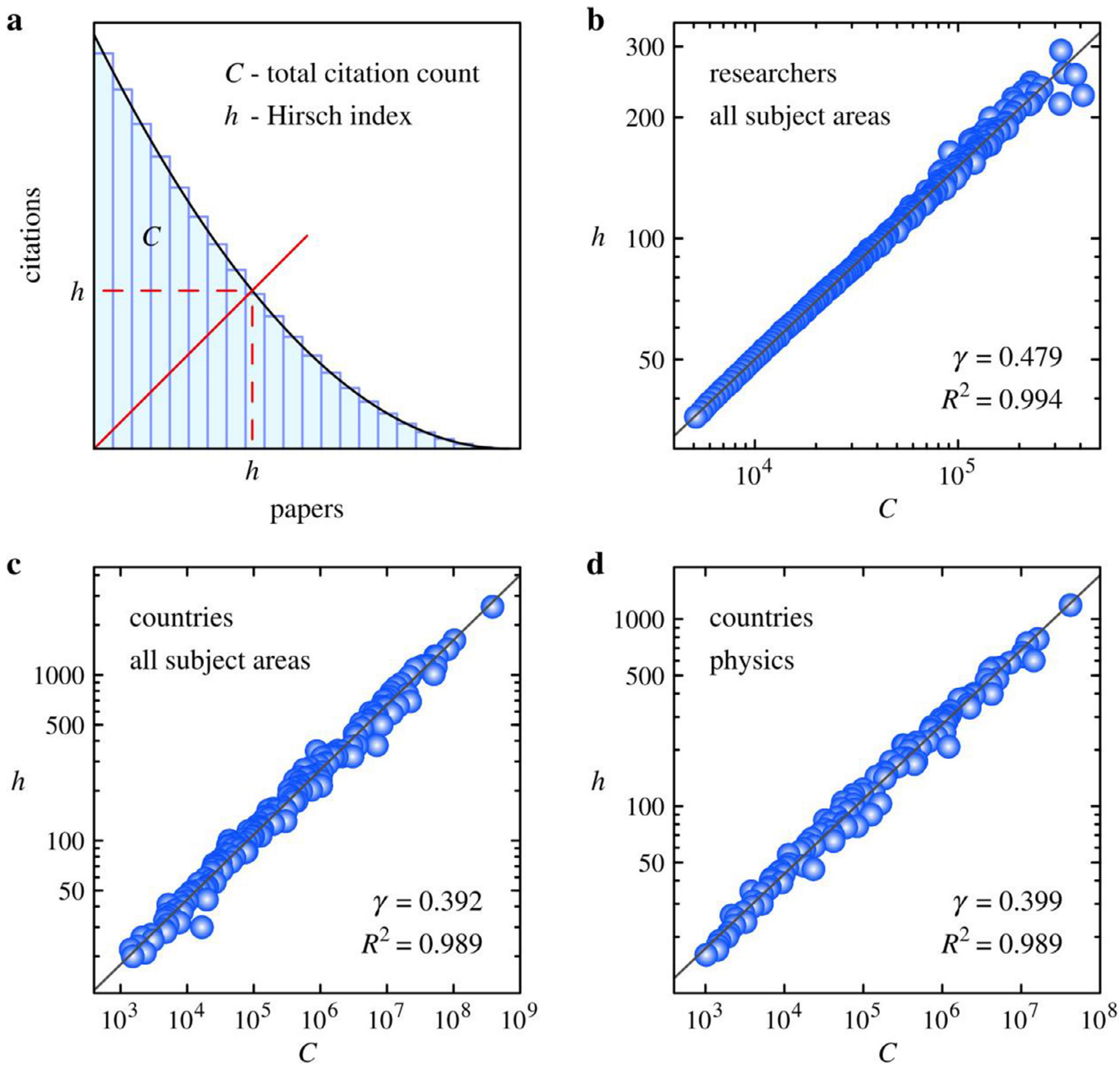

Fig. 1. Power laws for citation indicators. (a) A schematic rank-citation profile representing the numbers of citations to papers ranked in decreasing order and its continuous representation demonstrating the total citation count C (area under the curve) and the h-index. (b) A log-log plot of the h-index against the (averaged) total citation count C accumulated in 1996–2020 by individual researchers representing all subject areas. The grey line provides a linear fit. (c) The same as (b) but for research outputs of countries; C and h for a country are calculated based on papers which authors are affiliated with that country. (d) The same as (c) but counting only citations to papers in journals ascribed to the subject area *Physics and* Astronomy (see Material and methods).

citation metrics) and determine its parameters for different disciplines; then, a functional dependence between these parameters would provide global characteristics of the citation dynamics while the remaining parameters would be local characteristics of subject areas.

To find a suitable citation law, we consider the rank-citation profile for a scholar or some other research actor (Fig. 1a) ranking papers in decreasing order by the number of citations they receive. The discrete structure of the profile can be replaced by a mathematically more convenient continuous approximation (Egghe & Rousseau, 2006). The form of the profile can be characterized geometrically *via* a power-law relationship between its area and a characteristic linear in the coordinates of the axes. Such type of power laws has been employed to determine the fractal dimension of microscopic (Mandelbrot et al., 1984) and macroscopic (Lovejoy, 1982) physical objects. The area under the rank-citation profile is the total citation count C. Measuring the rank-citation profile along the axes corresponds to the highest number of citations for a paper and the total number of publications. These characteristics (distribution tails) are subject to large fluctuations. Therefore, we take a measure along the first quadrant bisector, equivalent (up to a coefficient) to taking the h-index (Hirsch, 2005), a key metric capturing both the quantity and quality of publications.

Here, we take the power-law relationship

$$h = a \cdot C^{\gamma} \tag{1}$$

and put it under scrutiny to reveal citation characteristics. We analyze the relationships between the power-law parameters a and γ for researchers and countries in different science domains. Based on the empirical analysis we propose a representation of the citation dynamics by a global scale and a set of local exponents.

2. Literature review

Citation analysis is a key component of the science of science (Zeng et al., 2017; Fortunato et al., 2018) providing a working tool to characterize the knowledge structure, understand its underlying mechanisms and statistical rules. The analysis applies to both static and dynamic properties of science systems (Zeng et al., 2017). In particular, future trends can be predicted by modeling the citation dynamics. Different approaches are employed to map the current state and evolution of scientific research. For instance, network analysis can reveal potentially influential research topics (Gao et al., 2021), predict the citation dynamics of a paper based on authors' profiles (Nanumyan et al., 2020), assign credit to datasets (Zeng et al., 2020). Various citation models are developed to determine factors that drive research impact (He et al., 2018).

A very important feature is the ability to analyze the dynamics of science on different levels, to address the issue of science inhomogeneity. The non-uniform structure of science is manifested in pronounced difference among research contributions of nations (Bornmann et al., 2018). On the other hand, it is instructive to analyze the dynamics of science on the level of research fields (Bornmann & Haunschild, 2022) relevant to the problem of field-normalization of science indicators (Radicchi et al., 2008). Such analysis depends strongly on the classification of science into a discipline structure (Glänzel & Schubert, 2003). Different kinds of research actors – individual researchers, institutions or entire countries – can be analyzed but the results of their assessment do not reflect only quality but also size (Molinari & Molinari, 2008). In particular, citation indicators of countries incorporate citations for a large number of researchers with different productivities and impact. Therefore, the rank-citation profile for a country, a non-trivial combination of the rank-citation profiles for country's researchers, exhibits statistical properties differing from those for an individual researcher.

To make any quantitative predictions, informative measures of the research activities and results are of paramount importance. Various science and technology indicators derived from bibliographic databases are now in use and being constantly developed (Glänzel et al., 2019). Despite their limitations, they provide useful and relevant information for research evaluation and modeling. The total number of citations is one of the simplest and most obvious indicators. Among the others, the h-index (Hirsch, 2005) retains a special role due to its widespread use. It is easy to understand and calculate; it satisfies administrative needs for rapid assessment. In some sense, the h-index is a robust measure because it is not influenced by uncited papers and accumulation of citations by highly cited papers (Egghe & Rousseau, 2006; Zeng et al., 2017). However, it has significant limitations (Costas & Bordons, 2007, Glänzel et al., 2019). Some of them can be attributed to the fact that the number of citations is often a poor proxy for a paper's quality (Zeng et al., 2017). Among other problems, one can mention that the h-index is not particularly useful for cross-disciplinary comparisons, does not take into account multiple co-authorship, self-citations, and the age of publications, depends strongly on the duration of a scientist's career. To be fair, it is unreasonable to expect a single indicator to assess adequately such multifaceted phenomenon as research performance (Costas & Bordons, 2007). Anyway, there is now a profusion of alternative metrics (Zeng et al., 2017; Glänzel et al., 2019). What is important is that the simplicity of the h-index justifies its use for analysis of the structure and evolution of science. In particular, the h-index depends linearly on the career length, logarithms of the productivity and citation rates (Burrell, 2007; Guns & Rousseau, 2009).

A key question in the studies of the h-index is its relationship with other metrics. Therefore, a significant attention has been paid to the power law Eq. (1). Its particular form with $\gamma = 0.5$ was already proposed in the paper that introduced the h-index (Hirsch, 2005), based on a model of constant rates of publishing and citation:

$$h \sim \frac{\sqrt{2r}}{1+r} C^{1/2}, \tag{2}$$

where r is the ratio of the number of citations earned by a paper in a year and the number of papers published in a year. A similar square-root dependence of h on C can be derived within a more general model: Let us assume that the geometric form of all the rank-citation profiles is the same, *i.e.* a rank-citation profile can be produced from some universal profile by uniform expansion with a coefficient q. Being the area under the curve, C is proportional to q^2 whereas h is proportional to q; thus, C and h satisfy Eq. (1) with $\gamma = 1/2$. Empirically, the distribution of $C^{1/2}/2h$ is sharply peaked around 1 (Redner, 2010). A square root relationship has been also suggested for h as a function of the number of publications N (Glänzel, 2006).

In general, γ can differ from $1/2$. The corresponding power-law relationship between C and h can be demonstrated within Lotka's model (Egghe & Rousseau, 2006). The model suggests a particular form of the rank-citation profile: the number of citations c to the paper with rank p is given by

$$c = kp^{-\alpha}. \tag{3}$$

The h-index defines a special point on the profile: if $p = h$ then c is also equal to h. Therefore, the coefficient k can be expressed as

$$k = h^{\alpha+1}. \tag{4}$$

The total citation count C is the sum of c over all the papers; it can be approximated by an integral:

$$C \sim \int_1^\infty c\mathrm{d}p = \frac{k}{\alpha - 1}. \tag{5}$$

The integration implies that $\alpha > 1$. Combining Eqs. (4) and (5) one gets a power-law relationship between C and h:

$$h = (\alpha - 1)^{1/(\alpha+1)} \cdot C^{1/(\alpha+1)}. \tag{6}$$

In this derivation, the power-law exponent is less than 1/2. Similar ideas can be employed to analyze tails of the rank-citation profile (Ye & Rousseau, 2010).

Eq. (1) has been tested successfully for different datasets (Van Raan, 2006; Radicchi & Castellano, 2013). Various power-law relationships have been proposed for the h-index. One example is based on exponents determined individually based on the rank-citation profile (Petersen et al., 2011). Another is the power law between N and h (Egghe & Rousseau, 2006); however, the empirical relation between the metrics is rather weak (Ye, 2009; Radicchi & Castellano, 2013). More complex power-law relationships involving all the 3 metrics h, C, and N have been proposed (Schubert & Glänzel, 2007) and shown to be rather accurate (Ye, 2009). However, it has been demonstrated on a large dataset that allowing for a power-law dependence of h on both C and N leads only to marginal improvement over Eq. (1) (Radicchi & Castellano, 2013). A number of power-law relationships are compared in Ref. (Malesios, 2015). The power-law model is not the only one; more complex predicting functions for the h-index have been proposed (Iglesias & Pecharromán, 2007; Bertoli-Barsotti & Lando, 2015). In the present study we are seeking not the most accurate but an adequate minimal model. Therefore, we limit the consideration by Eq. (1).

3. Material and methods

The numerical analysis in the paper is based on two sources of empirical data. First, citation and affiliation data for individuals are taken from an extensive database of most-cited researchers (Ioannidis et al., 2020) compiled according to a composite indicator from Scopus data for the years 1996–2020. The database provides an option to exclude self-citations from the metrics calculation; it also ascribes the researchers to 20 fields: *Agriculture, Fisheries & Forestry; Biology; Biomedical Research; Built Environment & Design; Chemistry; Clinical Medicine; Communication & Textual Studies; Earth & Environmental Sciences; Economics & Business; Enabling & Strategic Technologies; Engineering; Historical Studies; Information & Communication Technologies; Mathematics & Statistics; Philosophy & Theology; Physics & Astronomy; Psychology & Cognitive Sciences; Public Health & Health Services; Social Sciences; Visual & Performing Arts*.

Second, citation data for countries are taken from the SCImago Journal & Country Rank portal (SCImago, 2021), also based on Scopus. The portal provides scientific indicators which can be grouped by subject area (27 in total): *Agricultural and Biological Sciences; Arts and Humanities; Biochemistry, Genetics and Molecular Biology; Business, Management and Accounting; Chemical Engineering; Chemistry; Computer Science; Decision Sciences; Dentistry; Earth and Planetary Sciences; Economics, Econometrics and Finance; Energy; Engineering; Environmental Science; Health Professions; Immunology and Microbiology; Materials Science; Mathematics; Medicine; Multidisciplinary; Neuroscience; Nursing; Pharmacology, Toxicology and Pharmaceutics; Physics and Astronomy; Psychology; Social Sciences; Veterinary*. The subject areas are further partitioned into 311 subject categories. To define the subject areas and categories, the database employs the Scopus classification system for journals (Wang & Waltman, 2016). Citation indicators for a particular subject area or category are calculated based on citations to (but not necessarily from) papers in the journals associated with that science domain.

The citation data extracted from the databases are pairs of the h-index and the total citation count C for researchers and countries. In each dataset, all points with the same value of h are replaced by their average to mitigate the statistical noise. The geometric mean (arithmetic mean of the logarithms) of C is employed because the subsequent analysis is based on logarithmic measures; empirically, it results in somewhat better (in comparison with the arithmetic mean) description of the data by power laws, linearized by log-log plots of h against C. To reveal trends in the data, we limit the analysis to relatively high values of h, > 35 for individual researchers and > 15 for countries. The main difficulty in the choice of the cut-off values arises from strong differences among the research fields with respect to the ranges of C and h (and thus the numbers of data points). At the lower end of C and h, the power-law dependence (Eq. (1)) breaks in favor of higher values of C; the reasons can be different: insufficient averaging over researchers at low h in the case of countries and the use of a composite indicator to select entries in the dataset (Ioannidis et al., 2020) for individual researchers. The selected cut-offs try to maximize the amount of data without reaching the point where the power law breaks. For comparison, the numerical analysis has been repeated for slightly different values of cut-offs, 35±5 for individual researchers and 15±5 for countries. Although the sensitivity of parameters depends on the size of the research field, the results are rather robust with respect to the small changes in the cut-off values. The goodness of linear fits is estimated by the coefficient of determination R^2.

The following analysis is based on the coefficients and exponents of power laws for subsets of citation data. We consider only those subsets which have a significant amount of data, *i.e.* those with more than 25 data points (different values of the h-index). In the case of metrics for countries, all 27 subject areas but only 268 out of 311 subject categories are employed. For individual researchers, the condition is satisfied by 16 out of 20 subject areas (excluding *Communication & Textual Studies, Historical Studies, Philosophy & Theology, Visual & Performing Arts*), by 43 and 39 countries for metrics counting all citations and excluding self-citations, respectively.

4. Empirical power laws

The current study is based on Scopus, an extensive citation database of peer-reviewed literature. Instead of analyzing all the hundreds of millions of citations from scratch we employ the publicly available results of citation data processing for individual researchers (Ioannidis et al., 2020) and countries (SCImago, 2021). In addition, we bin the data by averaging all C for each value of h. First, we check whether the power law (Eq. (1)) holds empirically for researchers by linearizing it as

$$\ln h = \ln a + \gamma \cdot \ln C. \tag{7}$$

The corresponding plot (Fig. 1b) suggests that the power law is an adequate model to describe $h(C)$. C and h are co-varying measures; therefore, we determine the parameters γ and $\ln a$ by bivariate line fitting. In particular, we use standardized (also called reduced) major axis (SMA) estimation, appropriate for studies of power-law relationships (Warton et al., 2006); this type of estimation has been chosen based on its two advantageous properties: it is invariant with respect to interchange of the variables and their scaling. The exponent, $\gamma \approx 0.479$, is close to 0.5, as expected (see Literature review); some studies, however, suggest lower values of γ (Radicchi & Castellano, 2013; Van Raan, 2006). Elimination of self-citations (Appendix Fig. A.1) results in a power law with an exponent slightly closer to 0.5.

The ranges of C and h for individual researchers are rather narrow. Therefore, we turn to analysis of citation data for countries: their aggregate citation indicators reach rather high values $h \sim 2{,}500$ and $C \sim 4{\cdot}10^8$. Fig. 1c provides a log-log plot of h against C for countries, confirming that the power law (Eq. (1)) holds in different analytical contexts. The value of the exponent (~ 0.4) is significantly lower than that for researchers; we should notice that this particular value appears in the unification of major models for the *h*-index (Ye, 2011). Restriction of the citation data to the subject area *Physics and Astronomy* (Fig. 1d) has little effect on the power-law parameters. The same applies to *Mathematics* (Appendix Fig. A.2), a subject area with a low citation rate (Radicchi et al., 2008), which demonstrates that the power-law parameters are not determined by the average citation rate. One may presume that the subject area plays little role in the statistics. Actually, the variations between subject areas are significant (Appendix Table B.1): for instance, the $h(C)$ dependence for *Arts and Humanities* (Appendix Fig. A.2) has an exponent close to 0.5. Thus, the citation data restricted to subject areas provide an opportunity to analyze a non-trivial set of 27 closely related power laws.

5. Scales for countries

The citation data provide a set of exponents γ and coefficients a for subject areas. The idea is that these parameters for different areas are interrelated. To determine this relationship, we plot γ against $\ln a$ (Fig. 2a). Empirically, the scatter plot corresponds to a straight line (the goodness of fit $R^2 \approx 0.977$). Before addressing the question what this linear dependence means, we check its stability. Citation analysis for a shorter period of time (1996–2019 instead of 1996–2020) reveals a similar picture (Appendix Fig. A.3). This is not surprising because the two periods share the majority of citations. However, the main purpose of the analysis for 1996-2019 is to calculate the changes in $\ln a$ and γ due to citations received in 2020 (the corresponding data are provided in Appendix Table B.2). The question is whether minuscule $\Delta \ln a$ and $\Delta\gamma$ correlate with the data accumulated over a quarter century. A very simple model is that the linear dependence $\gamma(\ln a)$ is time-independent and the time evolution of subject areas results in moving the points along this line only. This model puts $(\Delta \ln a, \Delta\gamma)$ on the line passing through the origin with the same slope as the fit in Fig. 2a. Fig. 2b demonstrates that the citation data evolution is not far from this model, revealing intrinsic consistency of the results. For most of the subject areas, the changes are very small but a few of the areas are actively evolving. The data suggest a more complex behavior than convergence of the fields detected for the international research collaboration (Coccia & Wang, 2016).

What does an ideal linear dependence between $\ln a$ and γ mean for the power law (Eq. (1))? To address this question, we write the dependence as

$$\ln a = v - u \cdot \gamma. \tag{8}$$

Thus, for all pairs (a, γ), the following relation between v and u

$$v = \ln a + \gamma \cdot u \tag{9}$$

holds. Comparison with the logarithmic form of the power law (Eq. (7)) demonstrates that the point $(u, v) = (\ln C_0, \ln h_0)$ belongs to all the power laws for subject areas (see Fig. 2c for a schematic illustration). In particular, it means that a shift of the origin into the $(\ln C_0, \ln h_0)$ point (replacing $\ln C$ and $\ln h$ by $\ln(C/C_0)$ and $\ln(h/h_0)$) makes the graphs direct proportionalities, *i.e.*

$$\ln\left(h/h_0\right) = \gamma \cdot \ln\left(C/C_0\right) \tag{10}$$

or, equivalently,

$$\left(h/h_0\right) = \left(C/C_0\right)^{\gamma}. \tag{11}$$

In this equation, the parameters C_0 and h_0 are science-wide whereas the exponents γ depend on the subject area.

The form of Eq. (11) demonstrates that C_0 and h_0 provide scales for the total citation count and the *h*-index. The result is unconventional because power law functions epitomize scale invariance, a particular behavior under the dilatation transformation – scaling the argument causes only a proportionate scaling of the function itself. Each power-law dependence $h(C)$ is scale-invariant. Instead of C_0 and h_0 it determines only a $(= h_0 \cdot C_0^{-\gamma})$. However, taken together the power laws determine the scale because both C_0 and h_0 turn out to be global parameters. Going back from the global fit of Eq. (10) to the original fit $h(C)$ for a subject area, an error in $\ln a$ (parallel shift of the linear fit (Eq. (7)) is introduced, equal to the distance along the $\ln a$-axis between the corresponding point in Fig. 2a and the linear fit (Eq. (8)). The presence of a characteristic scale may result in different properties whether the system is above or below it. For instance, in the research collaboration dynamics the scale determines whether an effect of increase in self-reliance on the number of publications is positive or negative (Tokmachev, 2021). In the present case, the scale is relevant to comparison of

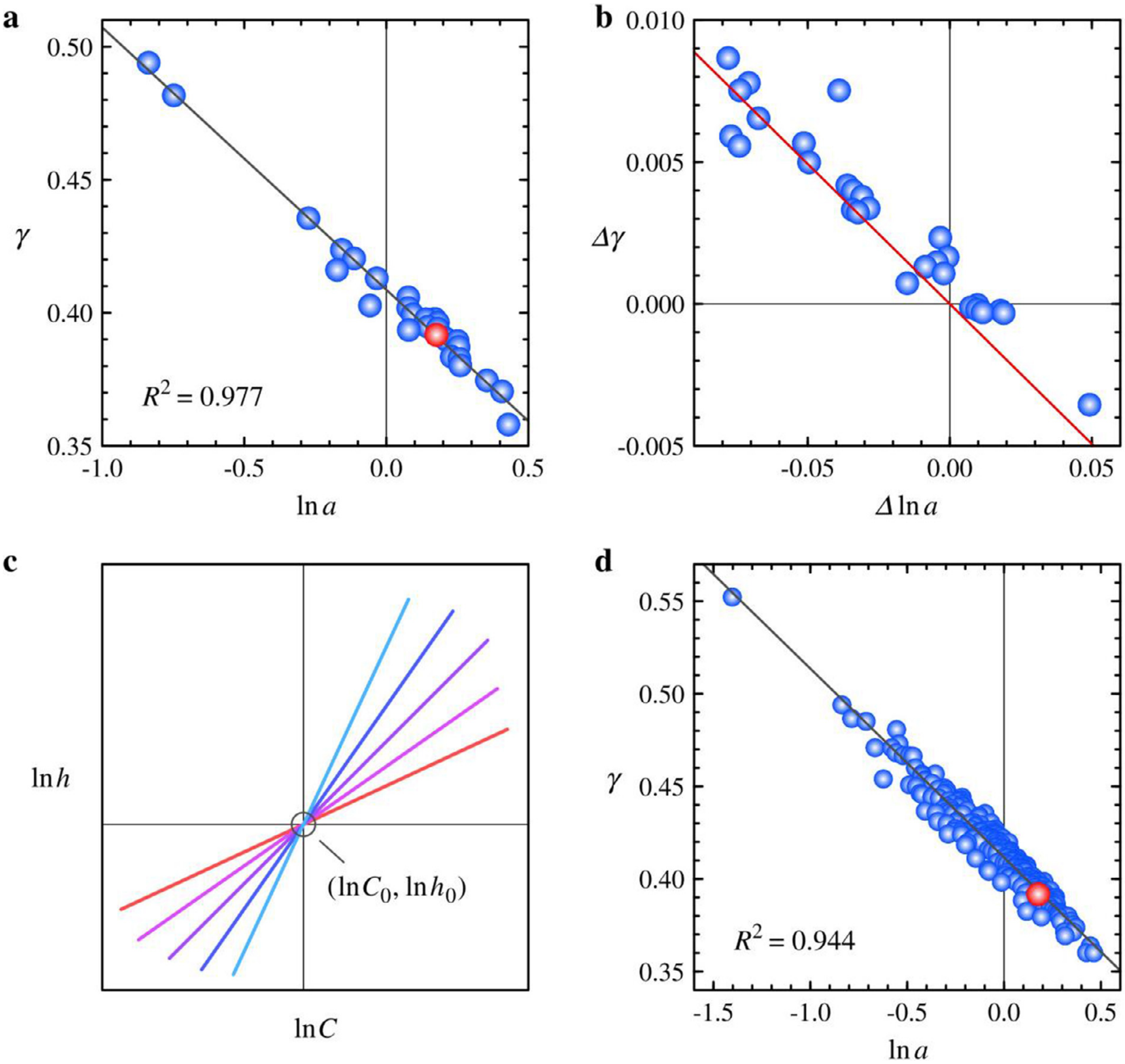

Fig. 2. Detection of hidden scales in citation statistics of countries (according to the affiliations of the authors of papers indexed in Scopus). (a) A scatter plot of the exponent γ against $\ln a$ for different subject areas. Each subject area generates a single blue dot; its coordinates are the parameters of the linear approximation (Eq. 7) between the logarithms of C and h accumulated by countries in 1996-2020. The grey line provides a linear fit of $\gamma(\ln a)$. The red dot corresponds to all subject areas taken together (Fig. 1c). (b) A scatter plot of changes in γ and $\ln a$ coordinates of dots in (a) due to citations received in 2020. The red line passes through the origin with the same slope as the fit in (a). (c) A schematic image of hidden scale emergence – plots of the h-index against the total citation count for different subject areas intersect in a single point (C_0, h_0). (d) The same as (a) but for 268 subject categories instead of 27 subject areas.

two subject areas (A and B) with different exponents ($\gamma_A > \gamma_B$): for the same value of C, we expect a higher h-index for the area A if $C > C_0$ but for the area B if $C < C_0$.

The numerical values of the scale parameters C_0 ($\approx$ 25,000) and h_0 ($\approx$ 63) seem reasonable: the point (C_0, h_0) lies well within the employed ranges of the C and h metrics (below the averages of (C, h) for 25 out of 27 subject areas). However, these values should be used with caution. The scale parameters come from exponentiation of u and v, numbers well above 1; as a consequence, small relative errors in the determination of u and v propagate into large relative errors in $C_0 = \exp(u)$ and $h_0 = \exp(v)$. Thus, logarithmic estimates of the scale parameters (Table 1) are more appropriate for discussion and error analysis. The errors in u and v stem partially from the inherent difficulties in assignment of citations to subject areas. The estimates with and without data binning differ by (slightly) more than 1 % (Table 1). Equation errors may also be important; after all, the power law is only a minimal model capturing the relationship between C and h.

Table 1
Parameters of hidden scales. The natural logarithms of scales for the total citation count (C_0) and the h-index (h_0) as well as the goodness-of-fit measures R^2 for plots of γ against $\ln a$. Data binning is applied (unless otherwise stated): in each dataset, all points (C_i, h) with the same value of h are replaced with a single point (C, h), where C is the geometric mean of C_i.

Scale determination	$\ln C_0$	$\ln h_0$	R^2
Countries			
by subject area, 1996-2020	10.14	4.14	0.977
(the same without data binning)	10.01	4.09	0.977
by subject area, 1996-2019	9.91	4.04	0.976
by subject category	9.81	4.04	0.944
by weighted subject category	9.89	4.07	0.964
Researchers (all citations)			
by subject area	8.91	3.77	0.961
by affiliation country	9.27	3.95	0.996
Researchers (without self-citations)			
by subject area	8.70	3.66	0.975
by affiliation country	9.09	3.85	0.998

To appraise the uncertainty in the scale values, we turn to an alternative division of the citation data into subsets by splitting subject areas into subject categories (SCImago, 2021). Fig. 2d presents a scatter plot of the exponent γ against $\ln a$ for subject categories – the data points line up to determine a scale of citation dynamics. The linear fit parameters $u = \ln C_0$ and $v = \ln h_0$ are rather close to those based on subject areas (Table 1) – the difference is about 3 % (more for C and less for h). Besides difficulties in ascribing citations to rather narrow subject categories, some part of this difference can be attributed to an uneven distribution of the categories among the subject areas. For instance, the area *Multidisciplinary* produces only one subject category whereas the area *Medicine* – 45. We can remove this non-uniformity artificially by applying weights to points in the plot of γ against $\ln a$ – all subject categories belonging to the same subject area are given equal weights summing to 1. For instance, each category of *Medicine* is assigned a weight of 1/45. The weights are employed for the SMA estimation and R^2. In both cases, the data points $(\ln a,\ \gamma)$ enter the calculation *via* sample means (such as variance). Therefore, the weighted estimation of the SMA linear fits and R^2 is implemented by replacing the ordinary arithmetic means $\sum_{i=1}^{n} a_i/n$ in the formulae with the corresponding weighted arithmetic means $\sum_{i=1}^{n} w_i a_i / \sum_{i=1}^{n} w_i$. This simple procedure eliminates about 30 % of the difference between the scales for subject areas and categories.

6. Scales for individual researchers

After establishing the scales for countries, we can test the approach on the data for individual researchers. We probe two schemes for citation metrics – counting all citations and excluding self-citations. First, we try the scale determination by division of the data into subject areas. Appendix Fig. A.4 illustrates the power-law dependence between C and h for a selection of natural sciences. The general problem is in a poorer description of the data by power laws (see Appendix Table B.3). Also, the number of datasets (subject areas) is smaller than for countries. Appendix Fig. A.5 shows the corresponding plots of the exponent γ against ln a; the values of the scales are provided in Table 1. We can see that the scales correlate with the characteristic values of C and h: the scale parameters for researchers are smaller than those for countries; exclusion of self-citations from the metrics reduces the scale parameters.

It is tempting to produce the scales by making a different, not based on subject areas or categories, division of the citation data. Geographical division would be a natural choice. In the case of statistics for countries, we can try it by ascribing countries to geographical areas and determining area-specific $\ln a$ and γ. Unfortunately, the range of γ (and $\ln a$) is too narrow to draw any reliable conclusions on the scale. Appendix Fig. A.6 illustrates this point employing division into 7 regions (Africa, America, Asiatic region, Eastern Europe, Middle East, Pacific region, and Western Europe). In the case of researchers, we can divide them into groups according to their country affiliations. Such division results in a larger number of data points and a more accurate linear fit of $\gamma(\ln a)$ than the subject-based division (see Fig. 3). As previously, exclusion of self-citations diminishes the scale parameters (Table 1). The parameters u and v determined by the two divisions differ by 4-5 %.

7. Discussion

In the literature, the empirical power laws of the form of Eq. (1) often focus on the exponents, especially in the case of the exponents being estimated parameters, not limited to integers or their simple ratios. A major reason to leave the coefficient out of discussion (such as replacing it with a direct proportionality symbol) is that in contrast to the exponents the coefficients have a dimensionality which is rather awkward for non-trivial exponents. Therefore, it is difficult to provide the coefficients with any meaningful interpretation. The reformulation of the power law in the form of Eq. (11) operating dimensionless scaled quantities solves the problem – it replaces the coefficient with the scaling parameters of the same dimensionality as the initial variables. The dimensionality argument validates the introduction of the scaling parameters C_0 and h_0 and suggests that the dimensionless form of

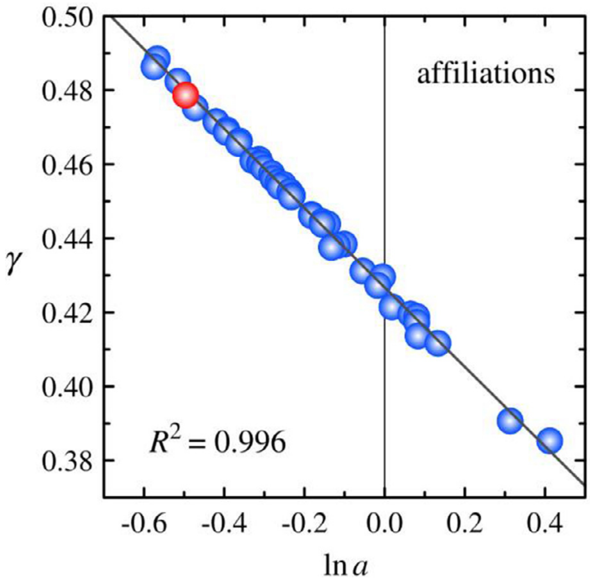

Fig. 3. Hidden scale based on affiliations of individual researchers. A scatter plot of the exponent γ against $\ln a$ for citations of researchers from different countries. Each country generates a single blue dot; its coordinates are the parameters of the linear approximation (Eq. 7) between the logarithms of C and h for the country's researchers. The grey line provides a linear fit of $\gamma(\ln a)$. The red dot corresponds to all countries taken together (Fig. 1b).

Eq. (11) should be rather common among power laws. A simple way to reduce a power law to its dimensionless form is by setting initial values. Taking into account the form of Eq. (11), this model would correspond to C_0 and h_0 being initial values for all subject areas evolving according to their γ's. However, the numerical values of C_0 and h_0 (given in Table 1) are too large to be considered initial values. Many actual (C, h) points, those with $C < C_0$ and $h < h_0$, cannot be reached from such initial values. Therefore, the initial value hypothesis is unlikely to be relevant to the emergence of global scale.

The structure of citation statistics is revealed by appropriate division of data into subsets. However, one should bear in mind that the procedure proposed above probes distribution of points around the global power fit and would fail in the case of ideal or close to ideal global power laws. The empirical analysis of citation dynamics demonstrates that the estimated scale parameters are rather close for different data divisions, by subject area or category for countries, by subject area or affiliation for individual researchers. One may presume that it manifests the existence of a global scale for each type of research actors to which different data divisions provide more or less reliable estimates. This is an attractive picture but there is little evidence to support it.

High value of R^2 can be rather deceptive. Appendix Fig. A.7 demonstrates a fan of linearized power laws (Eq. 7) based on the data of Appendix Fig. A.8 (affiliation-based division for researchers, excluding self-citations). This is the case with the highest value of R^2; yet the deviations from the ideal image of Fig. 2c are apparent. The image suggests a significant increase in the data variance as we move away from the crossing point. This property has been checked explicitly employing the full set of data. In practice, the numerical value of the scale can be varied significantly by division of the data into subsets. For instance, the removal of the data binning step from the estimate based on subject areas of researchers results in a 10 % decrease of u and v. In this case, the scale parameter h_0 goes below the analyzed range of h; the large difference between the scales can be associated with a low quality of power-law fits (R^2 as low as 0.43) for the non-binned data. Binning is less important for the estimates based on statistics for countries (Table 1) because the data are already averaged by the country affiliation. In general, the scale is not an inherent property of the citation data but depends also on the choice of the subsets. Therefore, it is advisable to consider the scale and exponents together, as a convenient representation of the citation dynamics for a particular division of science into domains. The emergence of global and local parameters suggests some parallels with the work (Radicchi et al., 2008) which describes the heterogeneity of the citation statistics by a global citation distribution curve and discipline-specific parameters – average citation rates. In the current study, the structural variations of citation patterns among the disciplines are described without any reference to citation rates.

It is desirable to get a coherent theoretical explanation of the empirical mathematical structure given by Eq. (11) and the numerical values of its parameters. One approach that can be probed is the mean-field approximation. However, it comes into conflict with the numerical values of C_0 and h_0 which are well below the mean values of C and h (as stated above, h_0 can even go below the analyzed range of h). Also, the mean-field model is difficult to reconcile with the fact that the changes in γ and $\ln a$ in a single year agree with the parameters for a period of 25 years (see Fig. 2b). Future studies should provide more information on the mechanism of the global scale formation. In particular, it would be interesting to carry out related studies employing other metrics. One of such metrics is h_α (van Eck & Waltman, 2008); it corresponds to measuring the rank-citation profile (Fig. 1a) not along the first quadrant bisector as in the case of standard h but along some direction defined by the parameter α. A study of the dependence between C and h_α for different values of α may shed light on the structure of the citation data.

The question is what conditions are necessary to claim the intrinsic character of the global scale, decoupled from the division of data? Surely, all the fits should be highly accurate and the data ranges should be broad. However, what would be particularly helpful is the presence of some natural parameter structuring the data. Such situation can be expected in physical phenomena. An example is provided by starch retrogradation and milk coagulation: more than half a century ago, it was noticed (Tuszyńsky et al., 1967) that plots of shear rate *vs.* shear stress for different moments of time meet at a common point (*cf.* Fig. 2c). Thus, the underlying mathematical structure is the same as for citations, leading to dimensionless variables related by equation similar to Eq. (11); the difference is that the exponents are not discrete parameters but continuous functions of time. A similar structure has been found experimentally and derived theoretically for Paris' law of crack growth (Carpinteri & Paggi, 2007). Finding a deeper connection between the scaled power laws in the two fields of research would benefit them both. The current situation is that the suggested theoretical derivation in continuum mechanics (Carpinteri & Paggi, 2007) cannot be transferred to explain Eq. (11) because the assumptions made have no direct analogy in citation analysis.

The proposed structure (Eq. (11)) associates subject areas with some characteristic exponents. The exponents provide a classification of the subject areas and deserve a special analysis. Take, for example, the data for countries (Fig. 2a). Most subject areas are rather similar – they have exponents close to 0.4; they also evolve towards higher values of γ (Fig. 2b). However, some subject areas differ significantly from the average. *Multidisciplinary*, gathering scientists and citations from various research fields, exhibits the highest value of γ. Incidentally, this subject area is known to not fit the model of a universal citation distribution (Radicchi et al., 2008; Waltman et al., 2011). Closeness of citation parameters for related areas may be an argument for their merge. For instance, the difference between the exponents for two economics-related subject areas, *Business, Management and Accounting* and *Economics, Econometrics and Finance*, is very small (Appendix Table B.1). In contrast, *Chemistry* and *Chemical Engineering* demonstrate a substantial difference between their characteristic exponents. The same applies to *Agricultural and Biological Sciences* and *Veterinary*, disciplines often bundled together (Abramo et al., 2013). In future studies, it would be interesting to track the evolution of the exponents characterizing research fields.

8. Conclusion

In summary, we offer empirical evidence of citation statistics representation by a global scale and local exponents for science domains. The work answers the call for domain-specific studies within the science of science (Fortunato et al., 2018). The representation stems from the power law between the total citation count and the *h*-index which holds for various domains, based on either subject or affiliation country. The determination of the power-law parameters for different disciplines and research actors provides insights into the inhomogeneity of science. It demonstrates the accuracy of the power-law model, reveals the influence of self-citations, the differences between the citation patterns of individual researchers and countries, the similarity or dissimilarity of research fields. It turns out that the parameters of the power laws are interrelated in such way as to determine scales for citation metrics. This underlying structure in the statistics of citation indicators solves the dimensionality problem for the coefficients in the power law $h(C)$, demonstrates that the statistics for a single year agrees with that for a quarter century, predicts the reversal of the order of disciplines (according to expected h for a given value of C) at the crossing point $(C_0,\ h_0)$. The visual representation of the collection of $h(C)$ relationships (Fig. 2c) suggests a way to correct the structural dissimilarity among disciplines by rotation in the $\ln C - \ln h$ plane employing the scale (u, v) as a pivot to collapse the fan of power laws into a single power law. The advantage of the proposed representation is that the characterization of science domains is single-parametric which facilitates studies of their evolution and classification. The results may inform future research policy decisions whereas the proposed approach to scale determination can be employed to understand the architecture of complexity in other power-law systems.

Declaration of Competing Interest

The author declares no competing financial interest or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Andrey M. Tokmachev: Conceptualization, Methodology, Formal analysis, Writing – original draft.

Appendix A

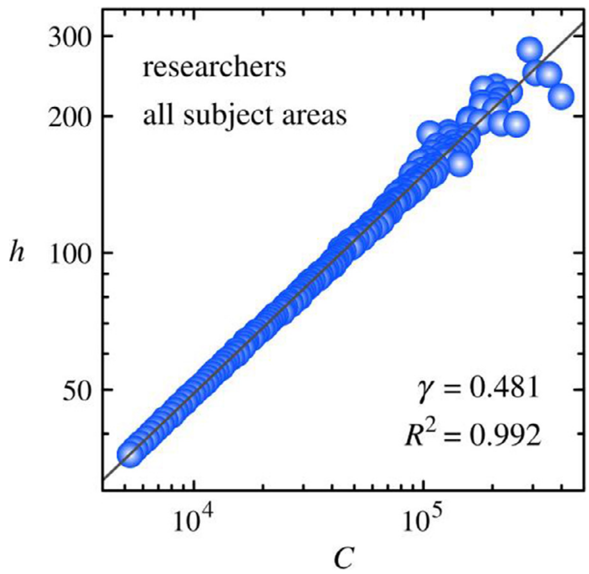

Fig. A.1. Citation statistics without self-citations. A log-log plot of the *h*-index against the (averaged) total citation count (both metrics exclude self-citations) accumulated in 1996–2020 by individual researchers representing all subject areas; the grey line provides a linear fit.

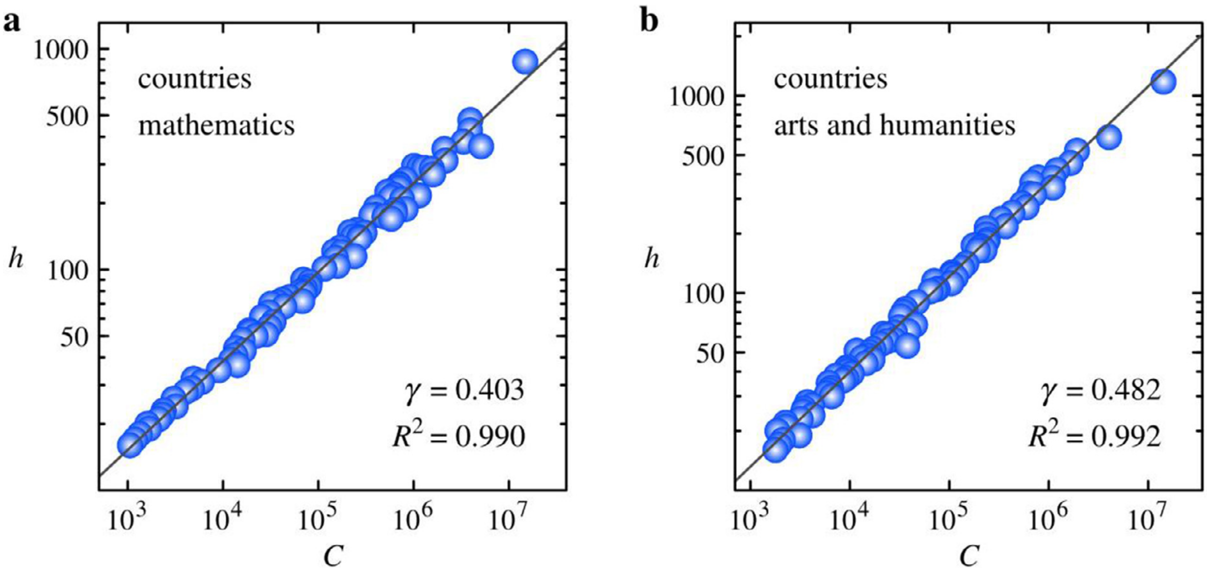

Fig. A.2. Log-log plots of the *h*-index against the (averaged) total citation count, based on citations received by countries in 1996-2020 for research outputs in (a) *Mathematics* and (b) *Arts and Humanities*. Grey lines provide linear fits.

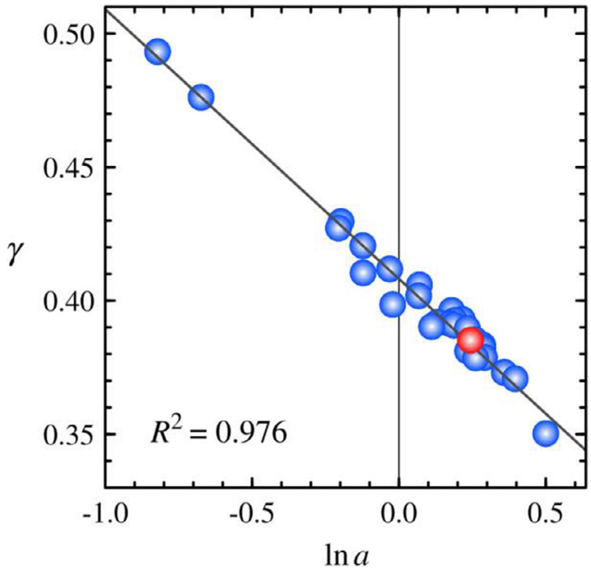

Fig. A.3. A scatter plot of the exponent γ against $\ln a$ for citation statistics of countries in different subject areas. Each subject area generates a single blue dot; its coordinates are the parameters of the linear approximation (Eq. 7) between the logarithms of C and h accumulated by countries in 1996–2019. The grey line provides a linear fit of $\gamma(\ln a)$. The red dot corresponds to all subject areas taken together.

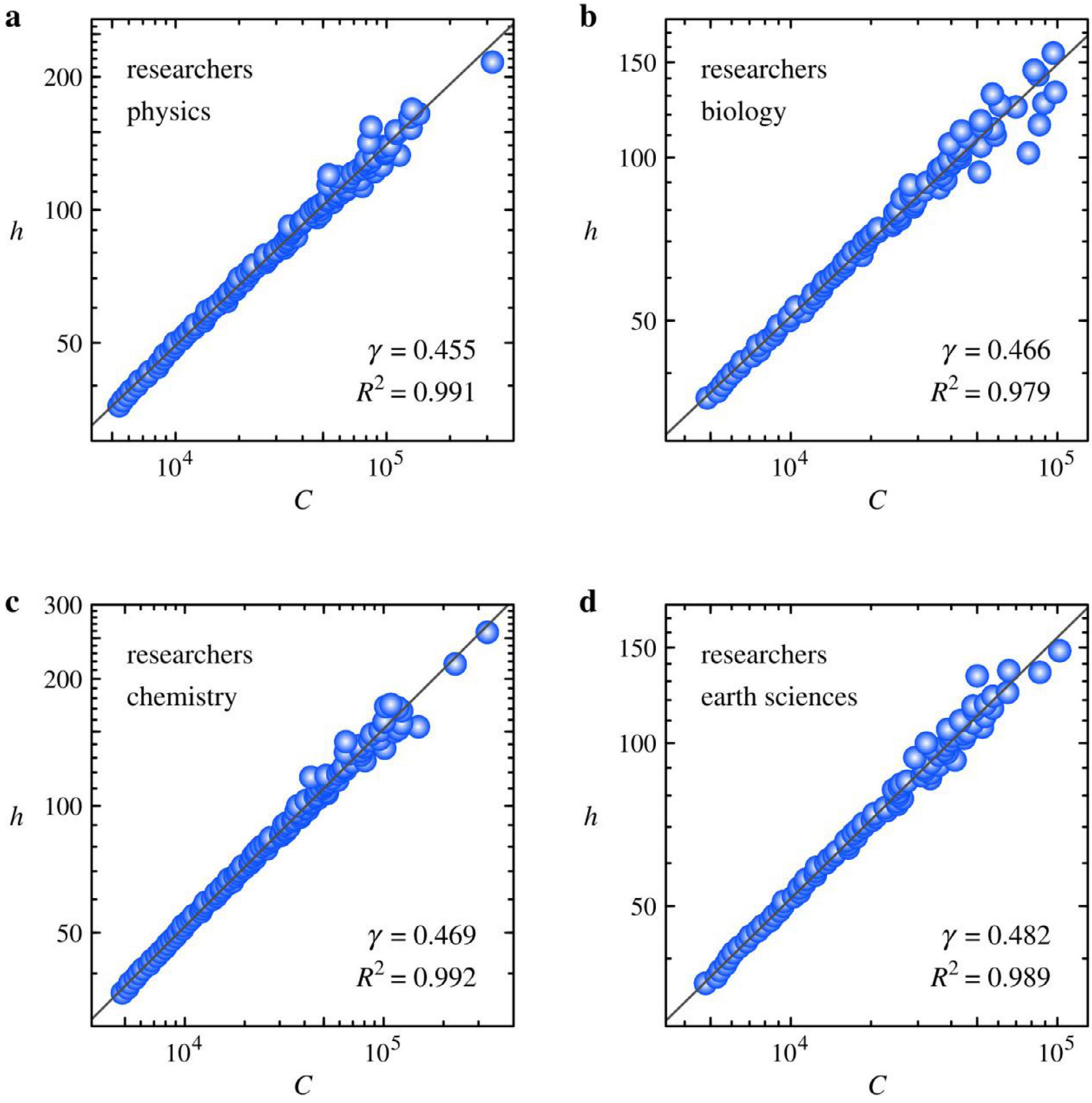

Fig. A.4. Log-log plots of the *h*-index against the (averaged) total citation count *C*, based on citations received in 1996–2020 by individual researchers in natural sciences: (a) *Physics & Astronomy*; (b) *Biology*; (c) *Chemistry*; (d) *Earth & Environmental Sciences*.

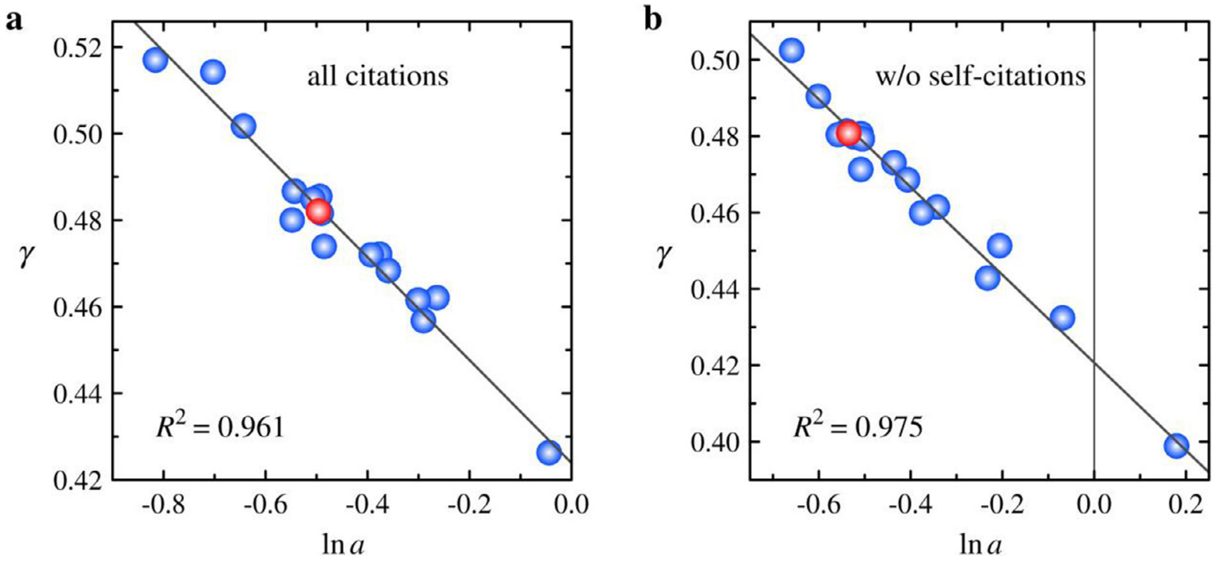

Fig. A.5. (a) A scatter plot of the exponent γ against $\ln a$ for citation statistics of individual researchers in different subject areas. Each subject area generates a single blue dot; its coordinates are the parameters of the linear approximation (Eq. 7) between the logarithms of C and h accumulated by individual researchers in 1996–2020. The grey line provides a linear fit of $\gamma(\ln a)$. The red dot corresponds to all subject areas taken together (Fig. 1b). (b) The same as (a) but excluding self-citations. The coordinates of the red dot are calculated based on the linear fit in Appendix Fig. A.1.

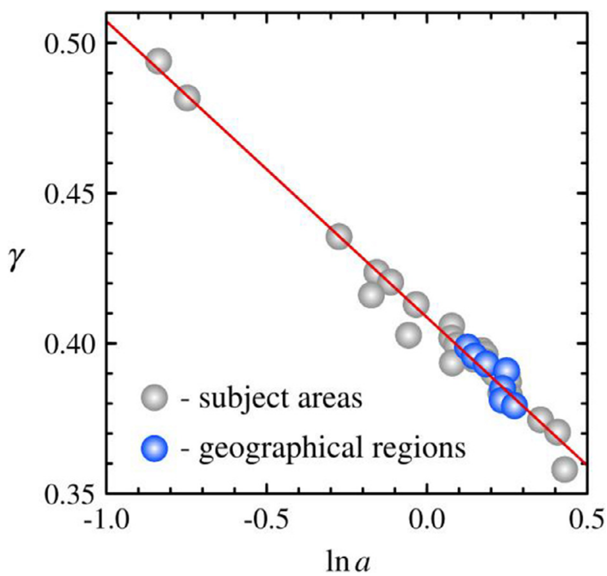

Fig. A.6. A scatter plot of the exponent γ against $\ln a$ for citation statistics of countries in different geographical regions. Each geographical region generates a single blue dot; its coordinates are the parameters of the linear approximation (Eq. 7) between the logarithms of C and h accumulated by countries of that region in 1996–2020; the data are superimposed on the data points (grey dots) and linear fit (red line) taken from Fig. 2a for subject areas.

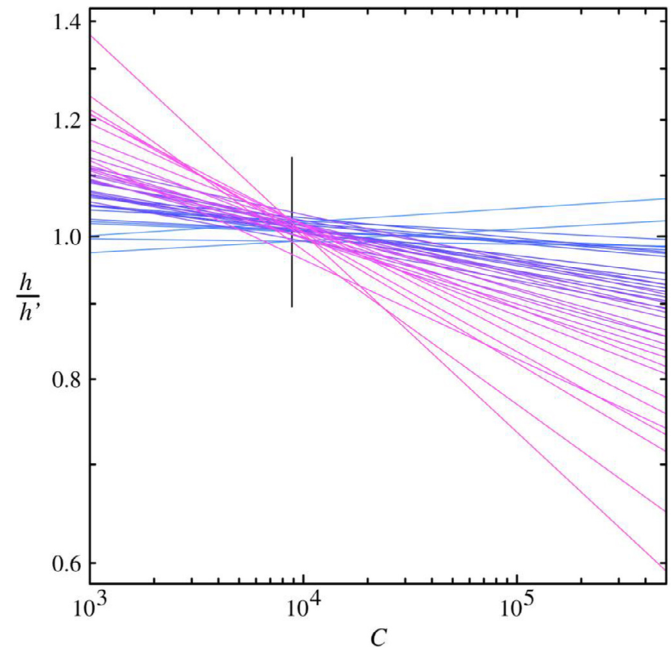

Fig. A.7. A log-log plot of lines fitting the dependences of h against C for citations (excluding self-citations) of researchers from different countries normalized by $h'(C)$, the global fit of Fig. A.1. The vertical line marks the scale parameter C_0.

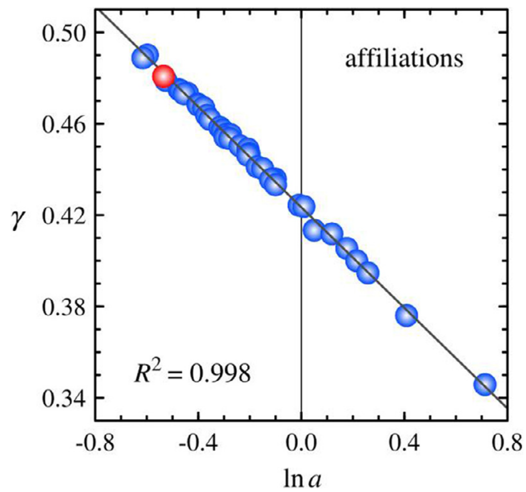

Fig. A.8. Hidden scale based on affiliations of individual researchers. A scatter plot of the exponent γ against $\ln a$ for citations of researchers from different countries, excluding self-citations. Each country generates a single blue dot; its coordinates are the parameters of the linear approximation (Eq. 7) between the logarithms of C and h for the country's researchers. The grey line provides a linear fit of $\gamma(\ln a)$. The red dot corresponds to all countries taken together (Appendix Fig. A.1).

Appendix B

Table B.1
Power-law parameters based on statistics for countries. The parameters γ and $\ln a$ of power laws between C and h of countries (Eq. (1)) based on citations received in 1996-2020 as well as the goodness-of-fit measures R^2 for the corresponding log-log plots.

Subject area	γ	ln a	R^2
Agricultural and Biological Sciences	0.375	0.354	0.989
Arts and Humanities	0.482	-0.748	0.992
Biochemistry, Genetics and Molecular Biology	0.389	0.251	0.990
Business, Management and Accounting	0.424	-0.157	0.989
Chemical Engineering	0.398	0.139	0.991
Chemistry	0.383	0.257	0.990
Computer Science	0.416	-0.173	0.988
Decision Sciences	0.436	-0.274	0.991
Dentistry	0.390	0.211	0.982
Earth and Planetary Sciences	0.394	0.181	0.992
Economics, Econometrics and Finance	0.420	-0.113	0.987
Energy	0.391	0.206	0.988
Engineering	0.383	0.230	0.987
Environmental Science	0.387	0.254	0.992
Health Professions	0.395	0.146	0.990
Immunology and Microbiology	0.398	0.171	0.991
Materials Science	0.380	0.260	0.989
Mathematics	0.403	-0.057	0.990
Medicine	0.402	0.077	0.988
Multidisciplinary	0.494	-0.837	0.987
Neuroscience	0.396	0.181	0.993
Nursing	0.406	0.077	0.990
Pharmacology, Toxicology and Pharmaceuticals	0.370	0.407	0.990
Physics and Astronomy	0.399	0.093	0.989
Psychology	0.413	-0.034	0.993
Social Sciences	0.394	0.079	0.992
Veterinary	0.358	0.429	0.973

Table B.2
Evolution of citation dynamics parameters for subject areas. Changes in the parameters γ and $\ln a$ of power laws between C and h of countries (Eq. (1)) due to citations received in 2020.

Subject area	Δγ	Δ ln a
Agricultural and Biological Sciences	0.00147	-0.0046
Arts and Humanities	0.00557	-0.0741
Biochemistry, Genetics and Molecular Biology	-0.00024	0.0179
Business, Management and Accounting	-0.00355	0.0492
Chemical Engineering	0.00499	-0.0495
Chemistry	0.00399	-0.0341
Computer Science	0.00567	-0.0514
Decision Sciences	0.00591	-0.0770
Dentistry	0.00752	-0.0738
Earth and Planetary Sciences	0.00866	-0.0780
Economics, Econometrics and Finance	-0.00020	0.0096
Energy	-0.00032	0.0190
Engineering	0.00233	-0.0034
Environmental Science	0.00378	-0.0309
Health Professions	0.00337	-0.0286
Immunology and Microbiology	0.00131	-0.0086
Materials Science	0.00165	-0.0008
Mathematics	0.00418	-0.0361
Medicine	-0.00003	0.0097
Multidisciplinary	0.00073	-0.0150
Neuroscience	0.00332	-0.0342
Nursing	-0.00012	0.0074
Pharmacology, Toxicology and Pharmaceuticals	-0.00030	0.0116
Physics and Astronomy	0.00752	-0.0390
Psychology	0.00107	-0.0022
Social Sciences	0.00322	-0.0323
Veterinary	0.00778	-0.0706

Table B.3
Power-law parameters for subject areas based on statistics for researchers. The parameters γ and $\ln a$ of power laws between C and h of individual researchers (Eq. (1)) as well as the goodness-of-fit measures R^2 for the corresponding log-log plots.

Subject area	γ	ln a	R^2
Agriculture, Fisheries & Forestry	0.460	-0.264	0.979
Biology	0.466	-0.359	0.979
Biomedical Research	0.469	-0.393	0.983
Built Environment & Design	0.509	-0.703	0.958
Chemistry	0.469	-0.376	0.992
Clinical Medicine	0.483	-0.543	0.995
Earth & Environmental Sciences	0.482	-0.494	0.989
Economics & Business	0.477	-0.548	0.948
Enabling & Strategic Technologies	0.481	-0.507	0.988
Engineering	0.497	-0.643	0.985
Information & Communication Technologies	0.471	-0.485	0.960
Mathematics & Statistics	0.426	-0.044	0.918
Physics & Astronomy	0.455	-0.291	0.991
Psychology & Cognitive Sciences	0.478	-0.491	0.970
Public Health & Health Services	0.459	-0.301	0.976
Social Sciences	0.512	-0.816	0.931

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